

Aristoxenus's Tunings: From Perceived Magnitudes to Analytical Applications

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At the May 2012 AAWM conference, perceptual aspects of Aristoxenus's tunings will be demonstrated by means of figures and sound files.

Passages in treatises by Cleonides and Aristides Quintilianus might lead one to believe that Aristoxenus's formulation of tuning several centuries earlier is well summarized by a model that comprises 144 pitch classes that correspond to a division of the octave into 144 equal intervals, each of which would correspond to the 144th root of 2, i.e., $2^{(1/144)} = \sim 8.33$ cents. However, a comprehensive account of Aristoxenus's *Harmonic Elements* leads to quite a different model. This model accords quite closely with recent outlooks and findings in music theory and cognition. Further, such a model of tuning could, in principle, be realized in practice when Aristoxenus wrote.

Instead of specifying values for each tuning that would correspond to 8 discrete, abstract, mathematical, area-less points among 145 such points (inclusive)

between two tones an octave apart, Aristoxenus understood pitches and pitch relationships to be perceptual and concrete. In particular, for Aristoxenus there could be more than one perceptual value for a particular pitch or interval in a tuning, i.e., a range of values, recently termed a 'pitch band' or 'interval span,' rather than a unique, crisp, all-or-nothing value. Accordingly, there might be higher and lower versions of a particular pitch and larger and smaller versions of a particular interval—just as in current European-derived discourse, one might distinguish a 'C sharp' from a 'sharp C,' or a small minor 2nd from a large minor 2nd.

Consistent with recent studies in experimental psychology, Aristoxenus claimed that the ranges of values for certain intervals were greater than for others. In particular:

Of the magnitudes of intervals, those of the concords [in modern terms, perfect 4ths, 5ths, 8ves, and their supplementary 11ths, 12ths, 15ths etc.] appear to have either no range of variation at all, being determined to a single magnitude, or else a range which is quite indiscernible, whereas

those of the discords [i.e., all other intervals] possess this quality to a much smaller degree.

The present report begins by showing how the 6 'familiar' tunings Aristoxenus describes could be realized by ear on a 7-stringed lyre. Like Aristoxenus, I presume only that a particular tone can be heard as the same in pitch as another tone and that a particular interval can heard as a concord or a discord, and as the same as or smaller than or larger than, another interval. Among the various possibilities, one tuning method stands out as being most readily applicable in actual practice as a result of requiring fewer steps in its realization.

In Aristoxenus's account, the relative sizes of the octave on one hand and on the other hand the perfect 4th and 5th are potentially problematic. At one point, he seems to treat their perceptual magnitudes as 'given.' Nonetheless, comments elsewhere in his treatise narrow the possibilities substantially. First, the stimulus counterpart of a perfect 4th would be, in abstract mathematical terms, between 2/5 and 3/7 (exclusive) the stimulus counterpart for a perfect 8ve (i.e., between 480 and ~514 cents if one assumes, but only for the sake of illustration, an octave of 1200 cents). Second,

the ways in which Aristoxenus's hemiolic-chromatic, soft-chromatic and enharmonic tunings would be most efficiently realized ensure that, again in abstract mathematical terms, the lower and upper limits of the perfect 4th's magnitude would correspond to 7/17 and 8/19 of the octave's magnitude (i.e., ~494.12cents and ~505.26 cents relative to an octave of 1200 cents). Third, in actual practice, Aristoxenus's subsequently controversial 'investigation' of his ongoing 'assumption' that the perfect 4th spans 5 semitones climaxes in a test that would narrow considerably, but not infinitely, the ranges of values that would qualify as concords.

Of relevance to Aristoxenus's investigation of this assumption, modern perceptual experiments have shown that there are 'individual differences' in the precision with which participants respond to particular interval stimuli. Similarly, Aristoxenus claimed not only that people differ in their auditory acuity but also that one can distinguish between competence and incompetence in aspects of perception relevant to his tuning formulation. All the same, one has to acknowledge that the climactic value of a perfect 5th in Aristoxenus's investigation need not correspond to a

single pair of abstract mathematical values for the perfect 4th and 5th that would be iterated 13 times. Instead, his investigation would succeed even if the final value were merely an average of 13 stimuli each of which was heard as a perfect 5th or perfect 4th.

Aristoxenus's account of tuning might seem to imply that the ability to distinguish from one another intervals as small as ~50, ~67, and ~75 cents was a reasonable criterion of musical competence. However, within the context of his main discussion of the 6 familiar tunings, one would only have to respond in this manner to intervals of twice this size, i.e., intervals of ~100, ~133, ~150, and ~200 cents. In any event, musicians trained in the Euro-American concert tradition arguably do so quite readily for intervals of ~100 and ~200 cents and musicians trained in classical traditions of North Africa, the Middle East, and Turkey do so for intervals of ~150 cents as well, as do classical musicians of Persia for intervals of ~133 cents.

Concerning musical analysis, Aristoxenus emphasizes that the *dynameis* (powers, capacities, abilities) of particular intervals are not merely a product of their magnitudes but are a result of their context: more

specifically, the strings a particular interval spans on a lyre. If the number of strings spanned is considered to correspond to the number of scale degrees spanned, one can characterize the tunings Aristoxenus formulates with regard to individual pairs of intervals that, to employ additional terms of recent music theory, constitute 'ambiguities' and 'contradictions.'

Aristoxenus's basic distinction between tense- and soft-diatonic tunings on one hand and, on the other hand, his chromatic and enharmonic tunings corresponds to a contrast between the latter, which comprise contradictions and the former, which do not. Among the non-contradictory diatonic tunings, the soft-diatonic is marked by ambiguity whereas the tense-diatonic is not. Among the contradictory chromatic and enharmonic tunings, there is an accumulation of ambiguous pairs of intervals that successively become contradictory pairs of intervals as one proceeds from the tonic-chromatic through the hemiolic- and soft-chromatic to the enharmonic. At the extreme ends of this spectrum, intervals of the enharmonic tuning would form distinctive ambiguities with intervals of the tense-diatonic tuning. Most important, ambiguities and contradictions are, in principle, audible as perceived

magnitude relationships between intervals spanning different numbers of degrees.

Consequently, in analyzing ancient Greek music, one could employ distinctions among 2nds, 3rds, etc. that, in principle, would be as perceptually significant as the distinctions between, e.g., minor 3rds and augmented 2nds in subsequent European-derived music theory. Moreover, beyond Aristoxenus's principal distinction between diatonic and chromatic/enharmonic (*pyknon*) tunings, one could acknowledge intervallic 'shades' or 'complexions' (*chroma* or *chroa*) that would audibly affect the networks of relationships between pairs of intervals within and between particular tunings.

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